

No Calculator

1. If $f(x) = x^2 + 2x - 4$, find $f(a)$, $f(a+h)$, and $\frac{f(a+h) - f(a)}{h}$

$$f(a) = a^2 + 2a - 4$$

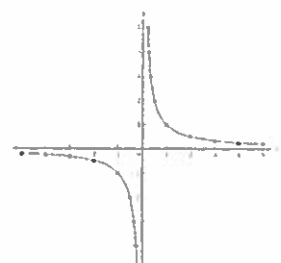
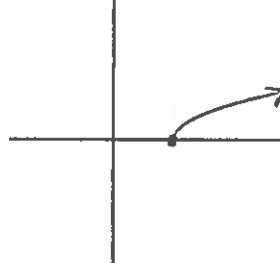
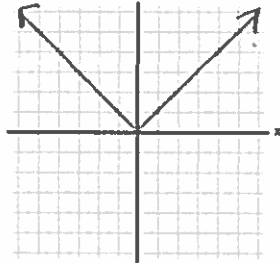
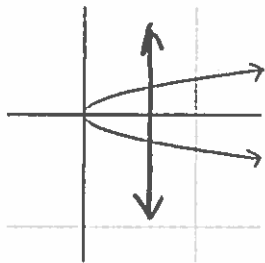
$$f(a+h) = (a+h)^2 + 2(a+h) - 4 = a^2 + 2ah + h^2 + 2a + 2h - 4$$

$$\frac{f(a+h) - f(a)}{h} = \frac{a^2 + 2ah + h^2 + 2a + 2h - 4 - (a^2 + 2a - 4)}{h} = \frac{h^2 + 2ah + 2h}{h} = h + 2a + 2$$

2. Which of the following figures are graphs of functions? State their domain and range.

Which of the functions are one-to-one? Which functions are even, odd, or neither?

- | | | | |
|----------------------------|-----------------------------|-----------------------------|---|
| A) | B) | C) | D) |
| Function: <u>No</u> | Function: <u>Yes</u> | Function: <u>Yes</u> | Function: <u>Yes</u> |
| One-to-one: _____ | One-to-one: <u>No</u> | One-to-one: <u>Yes</u> | One-to-one: <u>Yes</u> |
| Domain: $[0, \infty)$ | Domain: $(-\infty, \infty)$ | Domain: $[1.5, \infty)$ | Domain: $(-\infty, 0) \cup (0, \infty)$ |
| Range: $(-\infty, \infty)$ | Range: $[0, \infty)$ | Range: $[0, \infty)$ | Range: $(-\infty, 0) \cup (0, \infty)$ |
| Even or Odd: _____ | Even or Odd: _____ | Even or Odd: <u>Neither</u> | Even or Odd: <u>Odd</u> |



Reflects over y-axis

Reflects about origin

3. Suppose the graph of $f(x)$ is given. Describe how the graphs of the following functions can be obtained from the graph of $f(x)$.

- A) $y = f(x) + 8$ *shift $f(x)$ up 8*
- B) $y = f(x + 8)$ *shift $f(x) \leftarrow 8$*
- C) $y = 1 + 2f(x)$ *shifts $\uparrow 1$, stretches vert. by factor 2*
- D) $y = f(x - 2) - 2$ *shift $f(x) \rightarrow 2$ & $\downarrow 2$*
- E) $y = f(-x)$ *flips over y-axis*
- F) $y = -f(-x)$ *flips about the origin*
- G) $y = -f(x)$ *flips over x-axis*
- H) $y = f^{-1}(x)$ *flips over $y = x$ line*
- I) $y = f(x - 3) + 2$ *shift $f(x) \rightarrow 3$ and $\uparrow 2$*

4. Determine if the following are even, odd, or neither (without graphing). Show work.

A) $f(x) = \frac{x^2 - 3}{x^2 + 3}$

B) $f(x) = 3x^2 - 2x$

C) $f(x) = 5x^3 - x$

$f(-x) = \frac{(-x)^2 - 3}{(-x)^2 + 3} = \frac{x^2 - 3}{x^2 + 3}$ even

$f(-x) = 3(-x)^2 - 2(-x) = 3x^2 + 2x$ Neither

$f(-x) = 5(-x)^3 - (-x) = -5x^3 + x$ odd

5. If $f(x) = x^2 - 3x + 2$ and $g(x) = 4 - 3x$

A) Find $f \circ g$ $f(g(x))$

$$f(4-3x) = (4-3x)^2 - 3(4-3x) + 2$$

$$= 16 - 24x + 9x^2 - 12 + 9x + 2$$

$$= \boxed{9x^2 - 15x + 6}$$

B) Find $g \circ f$ $g(f(x))$

$$g(x^2 - 3x + 2) = 4 - 3(x^2 - 3x + 2)$$

$$= 4 - 3x^2 + 9x - 6$$

$$= \boxed{-3x^2 + 9x - 2}$$

6. Determine if $f(x)$ and $g(x)$ are inverses. (Show your work)

A) $f(x) = 3x - 2$ and $g(x) = \frac{1}{3}x + 2$

$$\frac{1}{3}(3x - 2) + 2$$

$$x - \frac{2}{3} + 2$$

No

B) $f(x) = \frac{1}{x-1}$ and $g(x) = \frac{1}{x} + 1$

$$\frac{1}{\frac{1}{x-1} + 1} = \frac{1}{\frac{1}{1} \cdot \frac{x-1}{1} + 1} + 1$$

$$= x - 1 + 1 = x \checkmark$$

yes

Calculator Allowed

7. If $f(x) = x^2 + 2x - 4$, find $f(x+1)$ and $f(\sqrt{2})$

$$f(x+1) = (x+1)^2 + 2(x+1) - 4$$

$$x^2 + 2x + 1 + 2x + 2 - 4$$

$$\boxed{x^2 + 4x - 1}$$

$$f(\sqrt{2}) = (\sqrt{2})^2 + 2(\sqrt{2}) - 4$$

$$2 + 2\sqrt{2} - 4 = \boxed{-2 + 2\sqrt{2}}$$

8. Find the domain and range of the following (You may want to look at a graph)

A) $f(x) = \sqrt{x+3}$ $x+3 \geq 0$
 $x \geq -3$

Domain $[-3, \infty)$
 Range $[0, \infty)$

B) $f(x) = \frac{2}{x+1}$ $x \neq -1$
 $y \neq 0$

Domain $(-\infty, -1) \cup (-1, \infty)$
 Range $(-\infty, 0) \cup (0, \infty)$

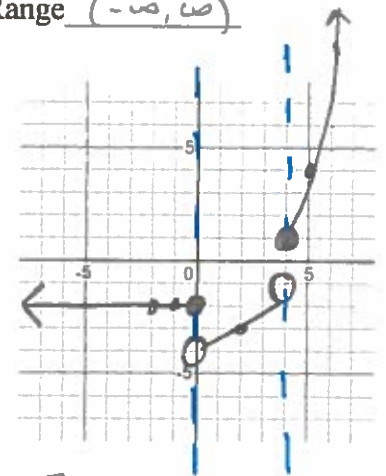
C) $f(x) = 7x + 15$ line

Domain $(-\infty, \infty)$
 Range $(-\infty, \infty)$

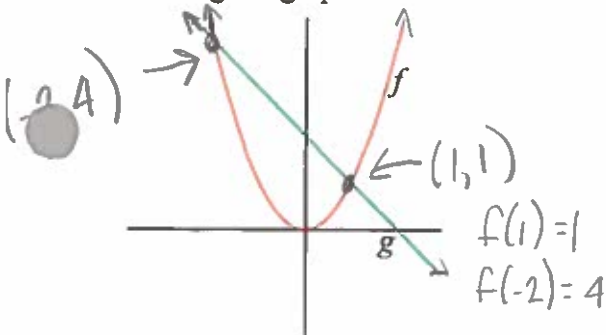
9. Graph the function

$$f(x) = \begin{cases} -2 & \text{if } x \leq 0 \\ \frac{1}{2}x - 4 & \text{if } 0 < x < 4 \\ (x-3)^2 & \text{if } x \geq 4 \end{cases}$$

X	Y	Notes
0	-2	closed
-1	-2	
-2	-2	
0	-4	open
2	-3	
4	-2	open
4	1	closed
5	4	
6	9	



10. Using the graph below...



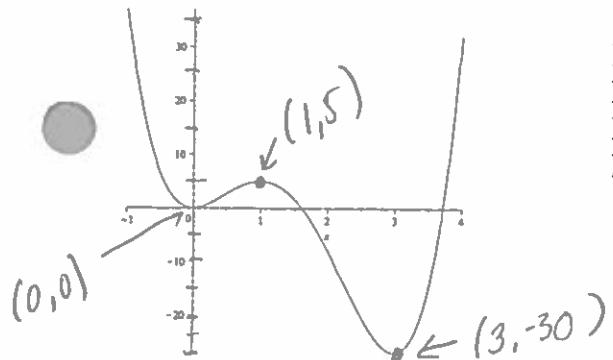
x values

- a) Where is $f(x) \leq g(x)$ $[-2, 1]$
- b) Where is $f(x) > g(x)$ $(-\infty, -2) \cup (1, \infty)$
- c) Where is $f(x)$ increasing? $(1, \infty)$
- d) Where is $g(x)$ decreasing? $(-\infty, \infty)$
- e) What is the net change of f , for $x = -2$ and $x = 1$?
- f) What is the average rate of change for f for $x = -2$ and $x = 1$?

$$f(1) - f(-2) = -3$$

$$\frac{-3}{1 - (-2)} = \frac{-3}{3} = -1$$

11. Given the function below find any Local Max (s): $(1, 5)$ or $y = 5$ upper humps
 Local Min (s): $(0, 0) + (3, -30)$ or $y = 0, y = -30$ lower humps
 Increasing Intervals: $(0, 1) \cup (3, \infty)$
 Decreasing Intervals: $(-\infty, 0) \cup (1, 3)$
 Is the function one-to-one? NO
 Even or odd? WHY Neither



not reflective over y-axis (even)
 not reflective about origin (odd)

12. Let $f(x) = 2x + 1$, find $f(a), f(h+a)$, the net change, and the average rate of change.

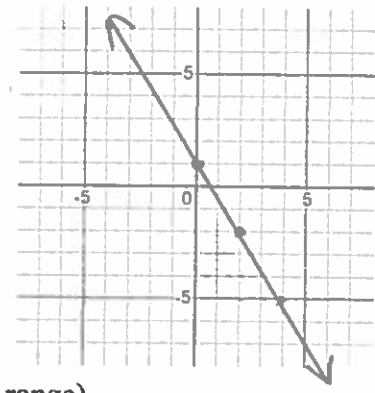
$f(a) = 2a + 1$ $f(h+a) = 2(h+a) + 1$ Net $\Delta = f(h+a) - f(a)$ Avg. $\Delta = \frac{2h}{h} = 2$
 $f(h+a) = 2h + 2a + 1$ $= (2h + 2a + 1) - (2a + 1)$
 $2h + 2a + 1 - 2a - 1$

$\text{Net } \Delta = 2h$

13. Let $f(x) = 3x^2 - 2x$, find the net change, and the average rate of change for $x = -4$ and $x = 2$.

$f(2) - f(-4)$ $f(-4) = 3(16) - 2(-4)$ $\text{Net } \Delta = 8 - 56 = -48$ Avg $\Delta = \frac{-48}{2 - (-4)} = -8$
 $48 + 8 = 56$

$f(2) = 3(4) - 2(2) = 12 - 4 = 8$

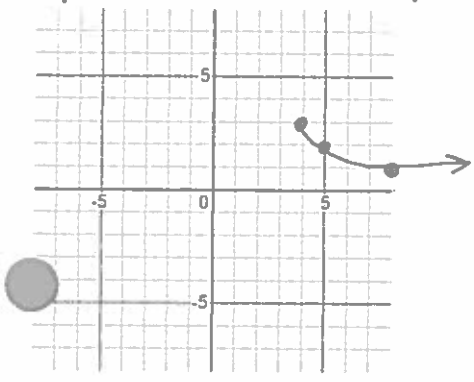


14. A) Sketch the graph $f(x) = 1 - \frac{3}{2}x$

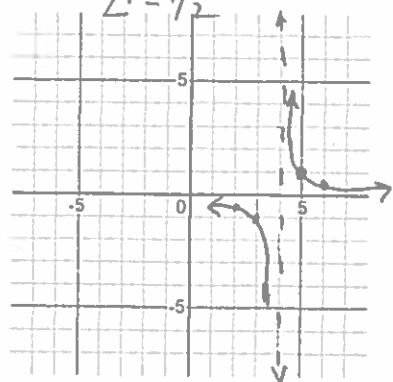
- B) what is the slope of the graph? $-3/2$
 C) What is the rate of change of the function? $-3/2$

15. Sketch the graphs of the following functions (state the domain and range)

A) $y = 3 - \sqrt{x-4}$ $x-4 \geq 0$ $x = 4$
 $x \geq 4$
 D: $[4, \infty)$
 R: $(-\infty, 3]$

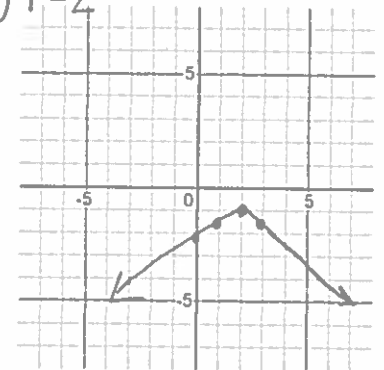


B) $g(t) = \frac{1}{x-4}$ $x \neq 4$
 $y \neq 0$



D: $(-\infty, 4) \cup (4, \infty)$
 R: $(-\infty, 0) \cup (0, \infty)$

C) $f(x) = -\frac{1}{2}|x-2| - 1$

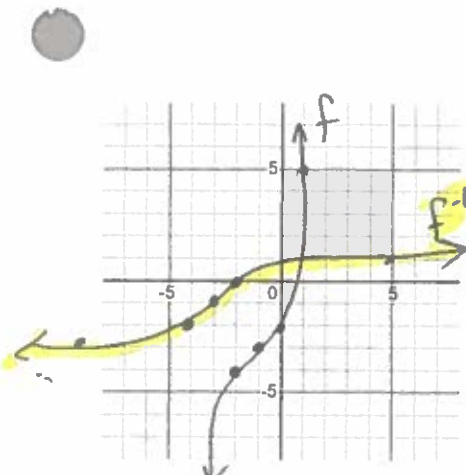


D: $(-\infty, \infty)$
 R: $(-\infty, -1]$

16. Find the inverse of the function and graph both.

A) $f(x) = (x+1)^3 - 3$ $f^{-1}(x) = \sqrt[3]{(x+3)} - 1$

B) $f(x) = \frac{2x+1}{3}$ $f^{-1}(x) = \frac{3x-1}{2}$



f		f ⁻¹	
X	Y	X	Y
-2	-4	-11	-3
-1	-3	-4	-2
0	-2	-3	-1
1	5	-2	0
5	11	5	1
-3	-11		

Switch

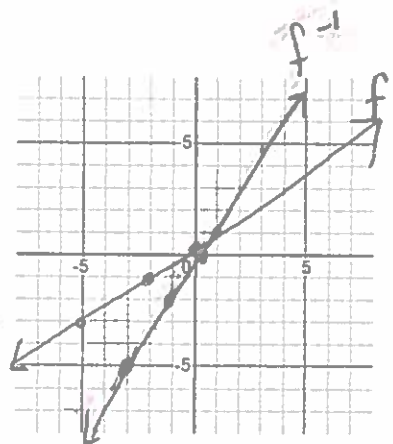
$$y = (x+1)^3 - 3$$

$$x = (y+1)^3 - 3$$

$$x+3 = (y+1)^3$$

$$\sqrt[3]{(x+3)} = y+1$$

$$\sqrt[3]{(x+3)} - 1 = y = f^{-1}$$



f		f ⁻¹	
X	Y	X	Y
-5	-3	-3	-5
-2	-1	-1	-2
0	1/3	1/3	0
1	1	1	1

$$y = \frac{2x+1}{3}$$

$$x = \frac{2y+1}{3}$$

$$3x = 2y + 1$$

$$3x - 1 = 2y$$

$$\frac{3x-1}{2} = y$$