24
125
9
1125

Acc. PreCalculus
Semester One End of Course Exam Review
No Calculator

Simplify the expressions.

1. $\left(\frac{1}{2}a^2b^3\right)^2(3a^{(-2)}b^{-1})^{(-3)}$

$\frac{1}{4}a^4b^6 \cdot \frac{1}{27}a^6b^3 = \boxed{\frac{a^{10}b^9}{108}}$

2. $(3a^2b)^{-2}\left(\frac{1}{5}a^{\frac{2}{3}}b^4\right)^3$

$\frac{1}{9a^4b^2} \cdot \frac{1a^2b^{12}}{125} = \boxed{\frac{b^{10}}{1125a^2}}$

3. Find the equation for the line that passes through the point (2, 4) and is perpendicular to the line $2x-4y+1=0$

$m = \frac{-2}{-4} = \frac{1}{2} \rightarrow m_\perp = -2$

$y = m(x-x_1)+y_1$

$y = -2(x-2)+4 \rightarrow \boxed{y = -2x+8}$

4. Find the equation for the line that passes through the point (4, –2) and is parallel to the line $5x+3y-2=0$

$m = \frac{-5}{3}$ $\quad y = -\frac{5}{3}(x-4)-2 \rightarrow y = -\frac{5}{3}x + \frac{20}{3} - \frac{6}{3} \rightarrow \boxed{y = -\frac{5}{3}x + \frac{14}{3}}$

Perform the indicated operation and simplify

5. $\frac{x^2y^2\left(\frac{x}{y}-\frac{y}{x}\right)}{x^2y^2\left(\frac{1}{x^2}-\frac{1}{y^2}\right)} = \frac{x^3y - xy^3}{y^2 - x^2} = \frac{xy(x^2-y^2)}{-1(x^2-y^2)} = \boxed{-xy}$

6. $\frac{\frac{5}{x-1}+\frac{-2}{x+1}}{\frac{1}{x-1}+\frac{1}{x+1}} \cdot \frac{(x-1)(x+1)}{(x-1)(x+1)} = \frac{5x+5+-2x+2}{x^2+x+x-1} = \boxed{\frac{3x+7}{x^2+2x-1}}$

Simplify the expressions.

7. $\frac{x^2-3x+2}{x^2-2x-3} \cdot \frac{x^2+5x+4}{x^2+2x-8}$

$\frac{(x-2)(x-1)}{(x-3)(x+1)} \cdot \frac{(x+4)(x+1)}{(x+4)(x-2)} = \boxed{\frac{x-1}{x-3}}$

8. $\frac{2x^2+3x+1}{x^2+2x-15} \div \frac{x^2+6x+5}{2x^2-7x+3}$

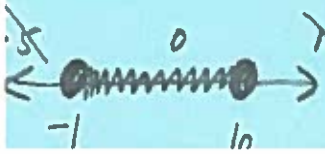
$\frac{(2x+1)(x+1)(2x-1)(x-3)}{(x-3)(x+5)(x+1)(x+5)} = \boxed{\frac{(2x+1)(2x-1)}{(x+5)^2} \text{ or } \frac{4x^2-1}{x^2+10x+25}}$

Solve the inequalities and state answer with INTERVAL notation.

9. $x^2-5x \le 6$

$x^2-5x-6 \le 0$

$(x-6)(x+1) \le 0$



$\boxed{[-1,6]}$

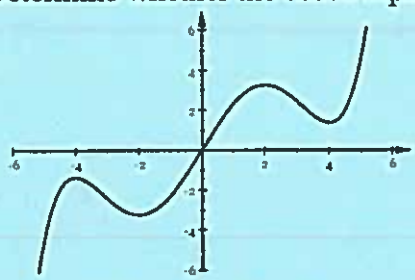
10. $x^2+3x > 10$

$x^2+3x-10 > 0$

$(x+5)(x-2) > 0$

$\boxed{(-\infty,-5) \cup (2,\infty)}$

11. Determine whether the curve represents a graph of a function and state why.



yes it's a function! passes the vertical line test :)

Determine whether the functions are even, odd, or neither

12. $f(x)=4-3x+2x^3-x^5$

$f(-x)=4+3x-2x^3+x^5$

Neither

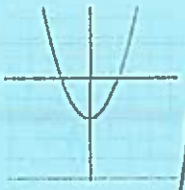
13. $f(x)=x^4+2x^2-3$

$f(-x)=x^4+2x^2-3$

Even

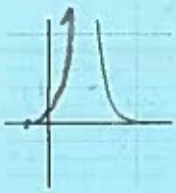
14. Determine which function(s) are one-to-one

A)



No

B)



No

C)

Yes

passes vert & horz line test

Find the inverse of the given functions

15. $g(x)=-x^2+2, \quad x\geq 0$

$x=-y^2+2$

$x-2=-y^2$

$2-x=y^2 \quad \pm\sqrt{2-x}$

but $y\geq 0$

$y^{-1}=\sqrt{2-x}$

16. $f(x)=2+\sqrt{x-4} \quad x\geq 4$

$x=2+\sqrt{y-4}$

$x-2=\sqrt{y-4}$

$(x-2)^2=y-4$

$y^{-1}=(x-2)^2+4$

$x\geq 2$

Find the domain of the functions

17. $f(x)=\dfrac{x}{x^2-3x+2}$

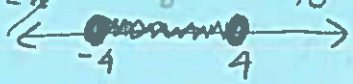
$(x-1)(x-2) \quad x\neq 1 \text{ or } 2$

$(-\infty,1)\cup(1,2)\cup(2,\infty)$

18. $f(x)=\sqrt{16-x^2}$

$16-x^2\geq 0$

$(4-x)(4+x)\geq 0$



$[-4,4]$

Express the quadratic functions in vertex form.

19. $f(x)=-3x^2-6x+7$

$-3(x^2+2x+1)+7+3$

$f(x)=-3(x+1)^2+10$

20. $f(x)=2x^2-12x-3$

$=2(x^2-6x+9)-3-18$

$f(x)=2(x-3)^2-21$

21. Find the polynomial of degree 4, with integer coefficients that has zeros of -i and 1, with 1 a double zero.

$i, -i, 1, 1$ (1 appears twice)

$(x-i)(x+i)(x-1)(x-1) \rightarrow (x^2+1)(x^2-2x+1) \rightarrow x^4-2x^3+2x^2-2x+1$

22. Find the polynomial of degree 4, with integer coefficients which has zeros of $-4, \frac{1}{2}$, and $2i$

$-4, 1/2, 2i, -2i$

$(x+4)(2x-1)(x-2i)(x+2i) \rightarrow (2x^2+7x-4)(x^2+4)$

$2x^4+8x^2+7x^3+28x-4x^2-16$

$2x^4+7x^3+4x^2+28x-16$

Describe the end behavior of the polynomials with proper notation.

23. $p(x)=2x^4-4x^3+7x$

degree of 4 lead coeff is positive

$\therefore$ $x\to+\infty, y\to+\infty$

$x\to-\infty, y\to+\infty$

24. $p(x)=-2x^3-x^2-5x$

degree of 3 lead. coeff is neg.

$\therefore$ as $x\to+\infty, y\to-\infty$

$x\to-\infty, y\to+\infty$

25. Determine the number of possible positive and negative real zeros using Descartes' Rule of Signs.

$p(x)=2x^4-5x^3+6x^2-100x+16$

$+\ -\ +\ -\ +$

$p(x)=+\ +\ +\ +\ +$

possible positive: 4, 2, or 0

possible negative: 0

26. Which of the following is a factor of the polynomial $P(x)=x^4-8x^3+17x^2+2x-24$.

(a) x + 4

(b) x + 2

(c) x + 5 ← not an option

(d) x + 1

	1	−8	17	2	−24
−4		−4	48	260	
	1	−12	65	262	

	1	−8	17	2	−24
−2		−2	20	−74	
	1	−10	37	−76	

	1	−8	17	2	−24
−1		−1	9	−26	24
	1	−9	26	−24	0

Find all asymptotes of the functions.

27. $r(x)=\dfrac{x-1}{x^2-3x-4}$ $\quad (x-4)(x+1)$

top deg 1 / bott deg 1 $\quad 1<2$

horz: $y=0$

Vert: $x=4$ & $x=-1$

Slant: None

28. $R(x)=\dfrac{x^2-3x-4}{x+2}$ $\quad (x-4)(x+1)$

top deg 2 / bott deg 1

	1	−3	−4
−2		−2	10
	1	−5	6

Horz: None

Vert: $x=-2$

Slant: $y=x-5$

29. List all the possible rational zeros of $f(x)=-2x^4+x^3-x^2-2x+6$.

$\dfrac{\pm 1, 2, 3, 6}{1, 2} = \pm 1, 2, 3, 6, \frac{1}{2}, \frac{3}{2}$

Evaluate the expressions and write in the form a + bi.

30. $(2-3i)^2$

$(2-3i)(2-3i)$

$4-12i+9i^2$

$4-12i-9 = -5-12i$

31. $\dfrac{5}{3-2i}\cdot\dfrac{3+2i}{3+2i}$

$\dfrac{15+10i}{9+4i^2} = \dfrac{15}{13}+\dfrac{10}{13}i$

Solve the equations in terms of ln.

32. $3+2e^{\frac{-1x}{2}}=9$

$2e^{-\frac{1}{2}x}=6$

$e^{-\frac{1}{2}x}=3$

$-\frac{1}{2}x=\ln 3$

$x=-2(\ln 3)$ or $\ln \frac{1}{9}$

33. $e^{\frac{-3x}{4}}=2$

$\ln e^{\frac{-3x}{4}}=\ln 2$

$-\frac{3}{4}x=\ln 2$

$x=-\frac{4}{3}\ln 2$ or $\ln\frac{1}{2^{4/3}}$ or $\ln\frac{\sqrt[3]{4}}{4}$

Solve the equations

34. $\log_2(x+1)+\log_2(x-1)=\log_2 8$

$\log_2(x^2-1)=\log_2 8$

$x^2=9$

$x=\pm 3$

$x=3$

35. $\log(x)+\log(x-3)=1$

$\log(x^2-3x)=1$

$x^2-3x=10^1$

$x^2-3x-10=0$

$(x-5)(x+2)$

$x=5$ & -2

$x=5$

36. Expand the logarithmic expression $\ln\left(\frac{x^3}{\sqrt[3]{2x-1}}\right)$ $\ln x^3 - \ln \sqrt[3]{2x-1}$

$3\ln x - 1/3 \ln(2x-1)$

37. Rewrite the following into a single logarithmic expression: $4\log x + \frac{1}{2}\log(x+2) - 3\log(x+5)$

$\log x^4 + \log \sqrt{x+2} - \log (x+5)^3 = \log\left(\frac{x^4\sqrt{x+2}}{(x+5)^3}\right)$

Evaluate the expressions

38. $\log_2 50 - \log_2 400$

$\log_2 \frac{1}{8} = \log_2 2^{-3} = -3$

39. $2\log 5 + \log 4$

$\log 5^2 \cdot 4 = \log 100 = \log_{10} 10^2 = 2$

40. Solve the system $\begin{cases} 2x+3y=-10 \\ y=3x-7 \end{cases}$ $(1, -4)$

$2x + 3(3x-7) = -10$

$2x + 9x - 21 = -10$

$11x = 11$

$x = 1$ $y = 3 - 7 = -4$

42. Evaluate the average rate of change of $f(x) = x^2 + 3x + 4$ between $x = -2$ and $x = 3$ $f(3) = 9 + 9 + 4 = 22$

$f(-2) = 4 - 6 + 4 = 2$

$\frac{f(3) - f(-2)}{3 - -2} \rightarrow \frac{22 - 2}{3 - -2} = \frac{20}{5} = 4$

43. Which point(s) are solutions to the inequality $y \le \frac{1}{2}x - 4$? Show work! or graph!

A) (0, -3) B) (5, -2) C) (7, 0) D) (9, 1) E) (2, -3)

$-3 \le 0 - 4$ F

$-2 \le 2.5 - 4$

$-2 \le -1.5$ T

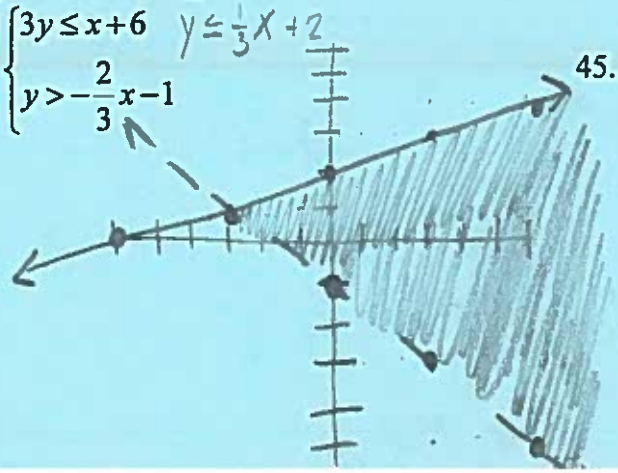
$0 \le 3.5 - 4$ F

$1 \le 4\frac{1}{2} - 4$ F

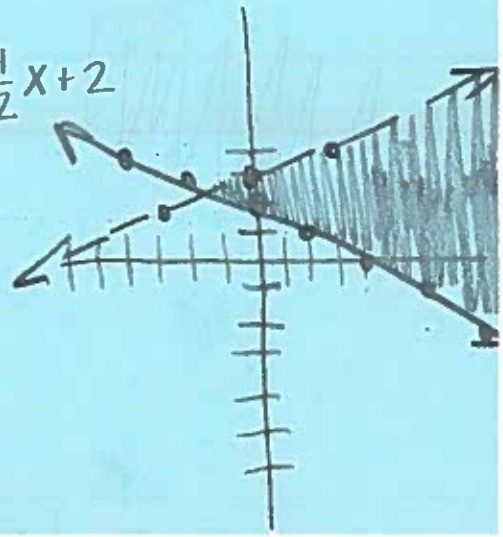
$-3 \le 1 - 4$ True

Graph the following.

44. $\begin{cases} 3y \le x+6 \\ y > -\frac{2}{3}x - 1 \end{cases}$ $y \le \frac{1}{3}x + 2$



45. $\begin{cases} 2y \ge -x+4 \\ y < \frac{1}{3}x + 3 \end{cases}$ $y \ge -\frac{1}{2}x + 2$



CHS Acc PreCalculus
End of Course Exam Semester One Review
CALCULATOR

Simplify the expressions and eliminate any negative exponent(s). Assume that all letters denote positive numbers.

46. $\sqrt[3]{2x^4y^2}\ \sqrt[3]{54x^2y^2}$

$2 \cdot 3^3$

$\sqrt[3]{2^2 \cdot 3^3 x^6 y^4} = \boxed{3x^2y\sqrt[3]{4y}}$

47. $\sqrt[4]{4x^2y}\ \sqrt[4]{4x^3y^5}$

$\sqrt[4]{2^4 x^5 y^6} = \boxed{2xy\sqrt[4]{xy^2}}$

Describe how the graphs can be obtained from the graph of f.

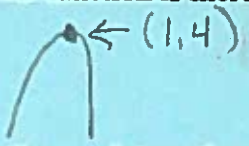
48. $y = -2f(x+2) - 3$

shifted left 2 down 3
flipped down and stretched vert. by a factor of 2.

49. $y = \frac{1}{2}f(x-1) + 4$

shifted → 1 & ↑ 4
shrunk vert by factor 1/2

50. Use a graphing device to draw the graph of $f(x) = 3 + 2x - x^2$. State approximately the interval(s) on which the function is increasing and on which the function is decreasing.

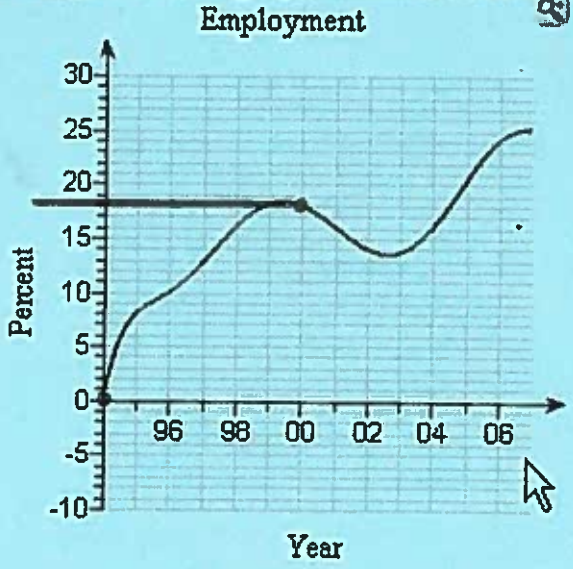


increasing: $(-\infty, 1)$
decreasing $(1, \infty)$

51. Use the graph to estimate the average rate of change of the percentage of new employees from 1994 to 2000.

$$\frac{f(2000) - f(1994)}{2000 - 1994}$$

$$\frac{18 - 0}{2000 - 1994} = \frac{18}{6} = \boxed{3}$$



If $f(x) = 2x - 1$ and $g(x) = 3x^2 + 2$ find

52. $(g \circ f)(x)$

$g(2x-1) = 3(2x-1)^2 + 2$

$3(4x^2 - 4x + 1) + 2 = \boxed{12x^2 - 12x + 5}$

53. $(f \circ g)(x)$ $f(3x^2+2) = 2(3x^2+2) - 1$

$= \boxed{6x^2 + 3}$

54. A ball is thrown vertically upward from the top of a 320-foot high building at a speed of 64 feet per second. How far above the ground is the ball at its highest point?

Use the formula $S(t) = S_0 + V_0 t - 16t^2$ initial height S_0(*feet*) and initial velocity V_0(*feet per* sec)

$S(t) = 320 + 64t - 16t^2$ calc. Max (2, 384)

$\boxed{384 \text{ ft}}$

Find all the zeros of the given polynomials.

55. $p(x)=x^3-x^2+12x-12$

$x^2(x-1)+12(x-1)$

$(x^2+12)(x-1)$ $x^2=-12$

$x=\pm 2i\sqrt{3}$ $\quad x=\pm 2i\sqrt{3},1$

56. $p(x)=2x^3-x^2+8x-4$

$x^2(2x-1)+4(2x-1)$

$(x^2+4)(2x-1)$

$x^2=-4$ $\quad x=\pm 2i,\ 1/2$

Find the domain, range, and asymptote if any of following.

57. $h(x)=e^{x-1}+2$

D: $(-\infty,\infty)$ R: $(2,\infty)$ A: $y=2$

58. $h(x)=-1+e^{x+2}$

D: $(-\infty,\infty)$ R: $(-1,\infty)$ A: $y=-1$

59. Brian wants to invest \$3,500 at an interest rate of 9.2% per year, compounded monthly. How long will it take his investment to grow to double? $A=P\left(1+\frac{r}{n}\right)^{(nt)}$

$7000=3500\left(1+\frac{.092}{12}\right)^{12t}$

$2=(1.0076666667)^{12t}$

$\frac{\log_{1.0076666667} 2}{12}=12t$ $\quad t=7.6$ years or 7 years 7 months

60. Approximately how long will it take an investment of \$500 to quadruple, if the interest rate is $5\frac{1}{4}$% per year compounded continuously? $A=Pe^{rt}$

$2000=500e^{.0525t}$ $\quad \ln 4=.0525t$ $\quad t=26.4$ years

61. Use a graphing device to find the solution of the equation. Correct to two decimal places. $3e^{x-1}=4-3x$

$y_1=3e^{x-1}$

$y_2=4-3x$ } find intersection (.64, 2.09) $\quad x=.64$

Find the solution of the equations

62. $\log_2(3x-1)-\log_2(2x)=2$

$\log_2\frac{3x-1}{2x}=2 \rightarrow \frac{3x-1}{2x}=4$ $\quad -1=5x$ $\quad \frac{-1}{5}=x$ $\quad \emptyset$

$3x-1=8x$

62. $\log_4(x+2)-\log_4(x-1)=\log_4 3$

$\log_4\frac{x+2}{x-1}=\log_4 3$ $\quad x=\frac{5}{2}$

$x+2=3x-3$ $\quad 5=2x$

Combine into a single logarithms

64. $\frac{1}{4}\ln(x^2+y^2)-3\ln(x-y)+2\ln(3x)$

$\ln\sqrt[4]{x^2+y^2}-\ln(x-y)^3+\ln 9x^2$

$\ln\frac{9x^2\sqrt[4]{x^2+y^2}}{(x-y)^3}$

65. $\frac{1}{2}\log(x^2+4x+4)-\log(x-2)+3\log(2y)^3$

$\sqrt{(x+2)(x+2)}$

$\log\frac{x+2}{x-2}+\log 8y^3=\log\left(\frac{8y^3(x+2)}{x-2}\right)$

65. Radium-123 has a half life of 500 years. Suppose we have a 200 g sample. How much of the sample remains after 300 years? $n(t)=n_0e^{(rt)}$

find r 1st. $\ln(1/2)=500r$ $\quad r=\frac{\ln(.5)}{500}$

$n(t)=200e^{r\cdot 300}$ $\quad n(t)=132.0$g

67. A radioactive substance has a half-life of 5 days. About how much of 125g sample will remain after 12 days?

find r 1st. $\ln(1/2)=5r$ $\quad r=\frac{\ln(.5)}{5}$

$n(t)=125e^{r\cdot 12}$ $\quad =23.7$g

68. The velocity of a sky diver t seconds after jumping is given by $v(t)=50(1-e^{-0.2t})$. After how many seconds is the velocity 30 ft/s?

$30=50(1-e^{-0.2t})$ $\quad .6=1-e^{-.2t}$ $\quad -.4=-e^{-.2t}$ $\quad .4=e^{-.2t}$ $\quad \frac{\ln .4=-.2t}{-.2}$

$t=4.6$ seconds

Solve the systems

69. $\begin{cases} x-y^2=-4 \\ x-y=2 \end{cases}$

$3: X-3=2 \quad X=5$

$-2: X+2=2 \quad X=0$

$-x+y=-2$

$-y^2+y=-6$

$0=y^2-y-6$

$(y-3)(y+2)$

$y=3 \quad y=-2$

$(0,-2)(5,3)$

70. $\begin{cases} x^2+y^2=25 \\ y^2=x+5 \end{cases}$

$-5: y^2=0 \quad y=0$

$4: y^2=9 \quad y=\pm 3$

$X^2+X+5=25$

$X^2+X-20=0$

$(X+5)(X-4)=0$

$X=-5,4$

$(-5,0)(4,3)(4,-3)$

Free Response:

71. $f(x)=\dfrac{-2}{(x-2)(x+4)}$ $\frac{-2}{-8}$ Graph showing all asymptotes, holes, and intercepts.

(a) Vertical asymptote(s) $X=2, X=-4$

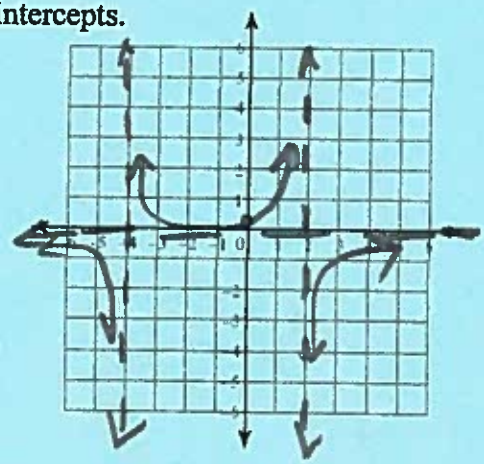
(b) Horizontal asymptote(s) $y=0$

(c) Hole(s) None

(d) y – intercept $1/4$

(e) x – intercept(s) none

(f) graph



72. Graph the system: $\begin{cases} x\geq 0 \ ① \\ y\leq 0 \ ② \\ y-x\leq -5 \ ③ \\ 3y\geq 2x-18 \ ④ \end{cases}$ If bounded state the vertices.

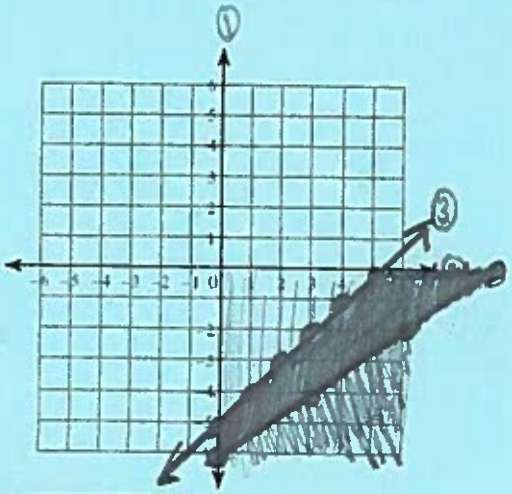
$y\leq x-5$ ③

$y\geq \frac{2}{3}x-6$ ④

$0\geq 2x-18$

$18=2x$

$9=x$



$(0,-5)(0,-6)(5,0)(9,0)$